Abstracts of the Talks at the 27^{th} Annual Meeting of the IMS

QUB, 5^{th}–6^{th} September 2014

Stephen Buckley (NUI Maynooth)

"Groups that are Isoclinic to Rings"

Let F be a finite algebraic system in which multiplication is denoted by juxtaposition. The *commuting probability of F* is

$$\Pr(F) = \frac{|\{(x,y) \in F \times F \ : \ xy = yx\}|}{|F|^2},$$

where $|\cdot|$ denotes cardinality. Much has been written on $\Pr(G)$ where G is a finite group. Together with MacHale and Ní Shé, we previously studied $\Pr(R)$ where R is a finite ring, making use of an associated notion of ring isoclinism.

In this talk, we compare and contrast the values attained by $\Pr(G)$ and $\Pr(R)$ as G and R range over certain classes of groups and rings, respectively. We show in particular that the set of values that arise for finite rings and for finite class-2 nilpotent groups are the same. Proving this involves the consideration of certain triples $T = (A, B, k)$ associated with both class-2 groups and rings. Isomorphism of such triples generalizes the previous notions of group isoclinism and ring isoclinism, and commuting probability of groups and rings is an isomorphic invariant of the associated triples.

This talk is based on joint work with Des MacHale (UCC).

Raúl Curto (University of Iowa)

"Truncated Moment Problems: The Interplay Between Functional Analysis, Algebraic Geometry and Optimization"

Inverse problems naturally occur in many branches of science and mathematics. An inverse problem entails finding the values of one or more parameters using the values obtained from observed data. A typical example of an inverse problem is the inversion of the Radon transform. Here a function (for example of two variables) is deduced from its integrals along all possible lines. This problem is intimately connected with image reconstruction for X-ray computerized tomography.

Moment problems are a special class of inverse problems. While the classical theory of moments dates back to the beginning of the 20th century, the systematic study of *truncated* moment problems began only a few years ago. In this talk we will first survey the elementary theory of truncated moment problems, and then focus on moment problems admitting cubic column relations.

For a degree $2n$ real d-dimensional multisequence $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{i \in Z^d_+, |i| \leq 2n}$ to have a representing measure μ, it is necessary for the associated *moment matrix* $M(n)$ to be positive semidefinite, and for the corresponding *algebraic variety*, V_β, to satisfy rank $M(n) \leq$ card V_β as well as the following *consistency condition*: if a polynomial $p(x) \equiv \sum_{|i| \leq 2n} a_i x^i$ vanishes on V_β, then $p(\beta) := \sum_{|i| \leq 2n} a_i \beta_i = 0$. In previous joint work with L. Fialkow and M. Möller, we proved that for the *extremal* case (rank $M(n) =$ card V_β), positivity and consistency are sufficient for the existence of a (unique, rank $M(n)$-atomic) representing measure.

In recent joint work with Seonguk Yoo we have considered cubic column relations in $M(3)$ of the form (in complex notation) $Z^3 = itZ + u\bar{Z}$, where u and t are real numbers. For (u, t) in the interior of a real cone, we prove that the algebraic variety V_β consists of exactly 7 points, and we then apply the above mentioned solution of the extremal moment problem to obtain a necessary and sufficient condition for the existence of a representing measure. To check consistency, one needs a new representation theorem for sextic polynomials in Z and $\bar{Z}$ which vanish in the 7-point set V_β.

Our proof of this representation theorem relies on two successive applications of the Fundamental Theorem of Linear Algebra. For other extremal moment matrices admitting cubic column relations, one can appeal to the Division Algorithm from real algebraic geometry to obtain similar representations; the Cayley–Bacharach Theorem also plays a role.

Stephen Gardiner (UCD)

"Universal Taylor Series"

In various mathematical contexts it is possible to find a single object which, when subjected to a countable process, yields approximations to the whole universe under study. Such an object is termed "universal" and, contrary to expectations, such objects often turn out to be generic rather than exceptional. This talk will focus on this phenomenon in respect of the Taylor series of a holomorphic function, and how the partial sums behave outside the domain of the function.

Gilles Godefroy (Paris VI)
"The Lipschitz-free Banach Spaces Associated with a Metric Space"

Let M be a metric space equipped with a distinguished point 0. Given such a pointed metric space M, the Lipschitz-free space $\mathcal{F}(M)$ over M is the linear span of the Dirac measures in the dual of the space $Lip(M)$ of Lipschitz functions on M which vanish at 0. This space $\mathcal{F}(M)$, which is actually a predual of $Lip(M)$, allows linearization of the Lipschitz maps between metric spaces, and its structure somehow reflects the properties of the metric space M. But although their definition is pretty simple, the Banach spaces $\mathcal{F}(M)$, which are separable when M is separable, are far from being well understood.

We will present some recent results e.g. on approximation properties of the free spaces, and natural open problems on this "new" class of Banach spaces.

Claus Koestler (UCC)
"Thompson Group F from the Viewpoint of Noncommutative Probability"

Recently we have given a characterization of all extremal characters of the Thompson group F which is motivated from our new proof of Thom's theorem about extremal characters of the infinite symmetric group. My talk will introduce our approach and address some open questions. This is joint work with Rolf Gohm.

Frank Lutz (Technical University Berlin)
"On the Topology of Steel"

Polycrystalline materials, such as metals, are composed of crystal grains of varying size and shape. Typically, the occuring grain cells have the combinatorial types of 3-dimensional simple polytopes, and together they tile 3-dimensional space.

We will see that some of the occurring grain types are substantially more frequent than others—where the frequent types turn out to be "combinatorially round". Here, the classification of grain types gives us, as an application of combinatorial low-dimensional topology, a new starting point for a topological mircostructure analysis of steel.

BRIGITTE LUTZ-WESTPHAL
(FREE UNIVERSITY BERLIN AND MATHEON)
"'Bringing Authenticity to the Mathematics Classroom"

Activities in the mathematics classroom should give an authentic feeling of doing mathematics. How can we close the gap between classroom mathematics and the fascination of mathematical research? Teachers and students need to learn to look at their own living environment with a mathematical eye. They have to get involved directly and individually in mathematical problems. Within the framework of learning in mutual dialogue they can start to act as little researchers. As "Math Investigators", students discover mathematics and are trained to ask questions with a mathematical content. We will report from our experience in a 4-year project "Mathe.Forscher" (Math Investigators) of the Stiftung Rechnen (German Numeracy Foundation).

GARY MCGUIRE (UCD)
"Online Security in Bits: Recent Developments in Public Key Cryptography"

All schemes and protocols in Public Key Cryptography today are based on one of two hard problems, the Integer Factorisation Problem or the Discrete Logarithm Problem. We will give a full introduction to these matters, including the historical development and connections between the IFP and the DLP. We will explain some recent developments in the DLP and their consequences. In particular, we will explain how polynomials of a certain shape were important in our results. Based on joint work with Faruk Gologlu, Robert Granger, and Jens Zumbragel. Our paper won the best paper award at the CRYPTO 2013 conference.

JAMES O'SHEA (NUI MAYNOOTH)
"Multiples of Pfister Forms"

Forms with diagonalization $\langle 1, a_1\rangle \otimes \ldots \otimes \langle 1, a_n\rangle$ for some scalars $a_1, \ldots, a_n$, known as Pfister forms, play a central role in the theory of quadratic forms. The sums and multiples of such forms in the Witt ring of a field have been subjects of much study, with Elman and Lam establishing long-standing results regarding their isotropy (non-trivial representations of zero).

We will consider the isotropy of multiples of Pfister forms over field extensions, establishing an improved lower bound on their Witt index (the dimension of a maximal totally-isotropic subspace). This bound enables us to add "maximal splitting" to the list of properties that are preserved under multiplication by a Pfister form. Conversely, in the case where the Pfister form is generated by variables, we will show that "going-down" results also hold with respect to multiplication.

GÖTZ PFEIFFER (NUI GALWAY)

"Computing with Groups and Algebras"

Many algorithms in Computational Group Theory follow the simple pattern of a breadth first search. I will discuss classical examples like the Todd–Coxeter procedure for coset enumeration in this context. The results of such algorithms can be regarded as graphs which frequently reveal surprising symmetries. Examples arise from finite Coxeter groups (like symmetric or dihedral groups), or more generally, from complex reflection groups. Linear versions of the algorithms can be used to compute with modules for finitely presented algebras. A cyclotomic Hecke algebra is a deformation of the group algebra of a complex reflection group that plays an important role in the representation theory of finite groups of Lie Type. I will present an application of a linear variant of the Todd–Coxeter algorithm to the cyclotomic Hecke algebras of some complex reflection groups which represents the algebra as a free module over a parabolic subalgebra. This establishes previously unknown cases of the freeness conjecture by Brou, Malle and Rouquier, which claims that a cyclotomic Hecke algebra, like the Iwahori–Hecke algebra of a finite Coxeter group, has a basis in bijection to the elements of the group.

ALAIN VALETTE (UNIVERSITÉ DE NEUCHATEL)

"The Kadison–Singer Problem"

In 1959, R.V. Kadison and I.M. Singer asked whether each pure state of the algebra of bounded diagonal operators on ℓ^2 admits a unique state extension to $B(\ell^2)$. The positive answer was given in June 2013 by A. Marcus, D. Spielman and N. Srivastava, who took advantage of a series of translations of the original question, due to C. Akemann, J. Anderson, P. Casazza, N.

Weaver,... Ultimately, the problem boils down to an estimate of the largest zero of the expected characteristic polynomial of the sum of independent random variables taking values in rank-one positive matrices in the algebra of n-by-n matrices. In turn, this is proved by studying a special class of polynomials in d variables, the so-called real stable polynomials. The talk will highlight the main steps in the proof.

Anthony W. Wickstead (Queen's University Belfast)

"Two-dimensional unital Riesz Algebras"

A *Riesz algebra* is an associative algebra over the reals that is simultaneously a vector lattice with the two structures connected by the implication $x, y \geq 0 \implies xy \geq 0$. My own interest is in *Banach lattice algebras* where there is also a complete norm that is compatible with the algebra and order structures in a very strict way. Although there are several nice examples, there is no general theory. In order to get a better understanding of the general situation, I decided to look at simple (but not particularly special) examples. Following Polya's dictum *If you can't solve a problem, look for a simpler problem that you can't solve.* which I combined with transfinite induction, I sought the simplest possible class of Banach lattice algebras that I didn't understand. I think I understand one-dimensional Banach lattice algebras, so I started thinking about two-dimensional Banach lattice algebras with an identity. Even these are varied enough to illustrate the kind of problems that a general theory of Banach lattice algebras will encounter. In this talk, I will describe the surprisingly rich family of all two dimensional unital Riesz algebras and (if time permits) start to explain why they are interesting to me.